Short Communication: DEA based auctions

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Abstract

In this paper we introduce a simulation framework which we use to numerically evaluate the Hybrid DEA - Second Score Auction. In a procurement setting, the winner of the Hybrid auction by design receives payment at the most equal to the second score auction. It is therefore superior to the traditional second score scheme from the point of view of a principal interested in acquiring an item at the minimum price without losing in quality. For a set of parameters we quantify the size of the improvements. We show in particular that the improvement depends intimately on the regularity imposed on the underlying cost function. In the least structured case of a variable returns to scale technology, the hybrid auction only improved the outcome for a small percentage of cases. However, for those few cases the improvement introduced by the hybrid auction is significant. For other technologies with constant returns to scale, the gains are considerably higher and payments are lowered in a large percentage of cases. In the simulations, we furthermore calculate the effect of the number of the participating agents, the concavity of the principal value functions, and the number of quality dimensions.

Keywords: Multi-dimensional auctions; Data envelopment analysis; Second score auction; Yardstick competition; Hybrid auction;

1. Introduction

The DEA - Second Score Hybrid auction introduced in the article entitled 'DEA based auctions' (see [1]) is qualitatively superior to the traditional Second Score Auction [2] (SSA). It shares the properties of being individu-

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ally rational, incentive compatible and socially optimal (allocatively efficient) with the SSA and at the same time, it may lower the costs to the procuring principal. The lower payment is made possible by exploiting the correlations or affiliations among the costs of the different agents.

In practice, however, the qualitative improvements over the SSA may not justify the added complexity of using a hybrid DEA-SSA auction. The quantitative improvements must be significant as well.

In this paper, we therefore introduce a framework for the simulation of such auctions and we compare the payment the winner of a hybrid auction receives to that of the winner of the standard second score auction. Moreover, we examine how payments are affected by the assumed cost regularities, the number of agents participating in the auctions, the concavity of the principal's value function and the agents' quality dimensions.

The rest of the paper is organized as follows: In Section 2 we analyse the DEA - Second Score hybrid auction, while giving some background in multi-dimensional auctions and DEA. In Section 3 we introduce the simulation framework and specifically the parameter spaces while we define the objectives of the simulations. In Section 4 we present the numerical results. In the first set of simulations, we analyse the Hybrid auction for single-dimensional quality (output) and compare Hybrid auction's payments under the use of VRS and CRS DEA technologies with the second score action. In the second set of simulations, we perform a similar analysis of the Hybrid auction but for a case of three-dimensional qualities. Finally in Section 5 we analyse our findings and draw conclusions as to their effects on the practical applications of hybrid auctions.

2. The DEA - Second Score Hybrid auction

The Hybrid auction introduces weak structure in the costs of producing different outputs. We consider a setting where different agents can produce different outputs at costs which are consistent with some underlying but unknown cost function. This cost function belongs to a broad class of cost functions, e.g. the set of all increasing and convex cost functions.

The Hybrid auction works by first assigning scores to the cost-output bids submitted by the agents. These scores are used to identify the agent with the highest potential to contribute to social welfare. That agent wins the auction and its payment is set to the minimum of the second-score payment and the DEA yardstick cost, based solely on the bids from other bidders.

To formalise, let us introduce a minimum of notation. Let the set of bidders be I, and let the output profile offered by bidder i be denote y^i. Also, let his reported, possibly manipulated cost be denoted x^i. The value function measuring the principal's benefit from different output profiles are denoted as $V(\cdot)$. Lastly, let the DEA estimated cost function based on all bids but the bid of bidder i be denoted $C^{DEA-i}(\cdot;k)$ where k is a parameter defining the DEA model used. In the $k=$ VRS case,

$$C^{DEA-i}(y;VRS) = min\{\sum_{j\neq i}\lambda^j x^j : y \leq \sum_{j\neq i}\lambda^j y^j, \sum_{j\neq i}\lambda^j = 1\}$$

where the last restriction is eliminated in the case of CRS.

The DEA - Second Score Hybrid auction runs as follow:

Step 1: The bidders submit price-output bids $(x^i, y^i),\ i \in I$.

Step 2: Each bid is assigned a score $S^i = S(x^i, y^i) = V(y^i) - x^i,\ i \in I$.

Step 3: The bid with the highest score wins, i.e. ignoring ties, the project is allocated to agent i when $S^i = S^{(1)}$

Step 4: The winner i is compensated with

$$b^i(x,y) = min\{C^{DEA-i}(y^i;k), V(y^i) - S^{(2)}\}$$

and losers are not compensated

In this outline, $S^{(1)}$ is the highest value of $S^i, i \in I$ and $S^{(2)}$ is the second highest value of $S^i i \in I$

In Figure 1 we provide the intuition behind the auction through a simple example. Assuming the use of VRS technology, the winner of the auction, agent 1, is paid a convex combination of agents' 2 and 3 bids.

For ease of future reference, let us call b above the Hybrid k payment, $V(y^i) - S^{(2)}$ the SSA payment and $C^{DEA-i}(y^i;k)$ the DEA k payment.

Like in the second score auction an agent's bid affects its chance of being selected, but not the compensation when it is selected. This is the key to the incentive compatibility. In addition to this, the use of benchmarking undermines the bidders' advantage of having private cost information. Through the use of a DEA model the equivalent of a second price outcome can be determined in contexts where the service bundles (i.e. the qualities or the outputs) offered by the different bidders are not entirely similar.

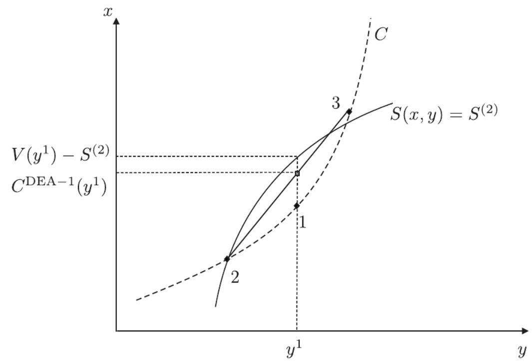

Figure 1: Graphical representation of the Hybrid auction.

3. Simulation Framework

Based on the Hybrid auction described in the previous section we now introduce the simulation framework. We consider a specific scenario in which the principal's concave value function is given by $V(y) = 4(1 - e^{-\alpha y})$ and the agents' cost function is given by $x(y) = cy^2$ for the VRS technology and $x(y) = cy$ for the CRS technology. The parameter c represents the agents' private information of their common unit costs and is independently drawn from the uniform distribution $U(0, 1)$. We will introduce multi-dimensional generalisations below. The value function has constant Arrow-Pratt coefficient of absolute risk aversion $r(y) = -V''(y)/V'(y) = \alpha$, and the more risk-averse the principal, the more convex (curved) is the value function, cf. eg. [3]. Lastly, the output level, often interpreted as the agents' quality level, is drawn from the uniform distribution $U(0, 2)$. In a given iteration, all agents face an underlying cost function of the same form, but their output levels and cost parameters differ.

We simulate the mechanism 10^4 times for a range of 3 to 60 agents. In every iteration we simulate the agents' costs and qualities (randomly drawn c and y), perform the selection of the agent with the highest score and record the payment it receives for Hybrid auctions using the VRS, and CRS DEA technologies and the second score auction. Due to the number of iterations we perform, the standard error in the mean values plotted is smaller than the symbol size in the plot (less than 10^{-3}) and thus we omit it for clarity.

Based on the above setting, we perform two sets of simulations. In the first set we examine the effect the concavity of the principal's value function has on the winner's payment (results detailed in plots in Figures 2 and 3), while in the second set we consider multi-dimensional qualities (Figure 4).

In both simulations we also explore the payments' sensitivity to the number of agents participating in the auction.

Technically, all simulations are done in R and all DEA programs are solved using the "Benchmarking" package for R, cf. [4] and [5]

4. Simulation Results

Having detailed the simulation's input parameters and objectives, we now present our numerical findings. The most notable result for both cases (single and multi-dimensional qualities) is that the expected payment for the winning agent depends intimately on the assumed regularity of the underlying cost function. With the least ex ante assumptions, in the Hybrid VRS auction, the payment is the same or almost identical to the second score auction. For the Hybrid CRS auction though, the payment is significantly less than the second score auction. This can be seen in Figures 2 and 3 (Plots a and b) for the single-dimensional quality and in Figure 4 (Plots a and b) for the multi-dimension quality. In the following sections we look at each case in greater detail and provide the intuition behind the main result.

4.1. Single-dimension output

Initially we fix the concavity of the principal's value function by $\alpha = 1$ and compare the payment the winner expects to derive in the Hybrid k auction ($min\{C^{DEA-i}(y^i;k), V(y^i) - S^{(2)}\}$) with the second score auction ($V(y^i) - S^{(2)}$) as the number of agents increases from 3 to 60. First, we use VRS technology (k=VRS) and simulate the costs using a quadratic cost function, while we then proceed to use the CRS technology (k=CRS), with costs being simulated by a linear cost function $x(y) = cy$. We show that as the number of agents increases the winner's average Hybrid VRS payment is almost equal to the second score auction payment (Figure 2 : Plot a), with the ratio between Hybrid VRS and SSA ranging from 0.98 to 0.99. For its liner cost counterparts, the Hybrid CRS payment is significantly lower than the expected payment in the second score auction (Figure 2: Plot b) with the ratio ranging from 0.6 to 0.27.

We proceed to study the Hybrid VRS auction in more detail by showing in Figure 2 : Plot c that for 82.9% to 92.25% of the iterations of the algorithm, the second score payment $V(y^i) - S^{(2)}$ is lower than the DEA VRS payment $C^{DEA-i}(y^i;k)$ (with k=VRS). Also, the simulations showed that hyper-efficiency, i.e. the possibility of VRS being unable to provide a cost

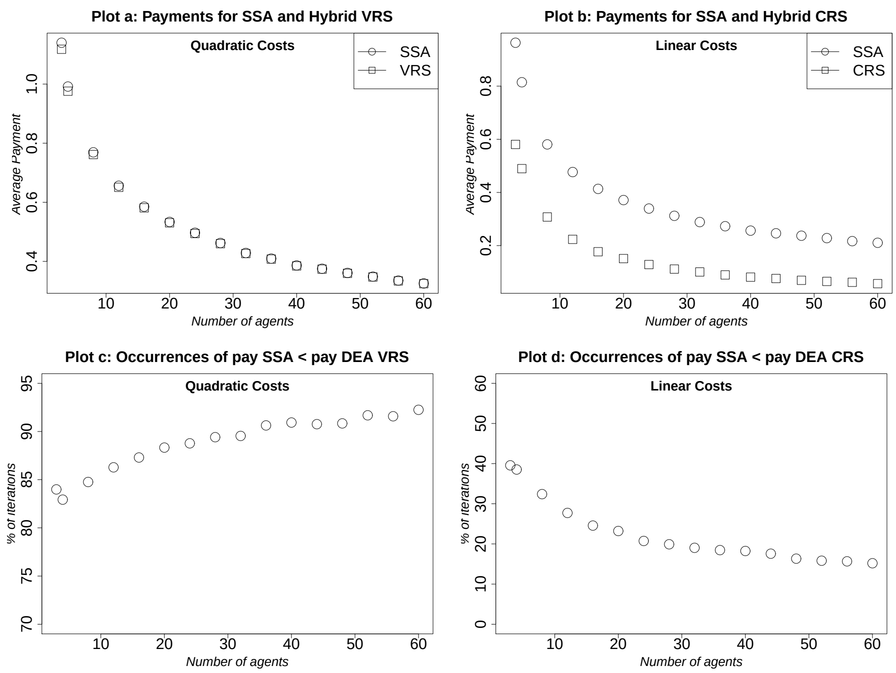

Figure 2: Single dimensional qualities for $n = \{3, ..., 60\}$ agents, $\alpha = 1$, $c \sim U(0, 1)$, and $y \sim U(0, 2)$.

norm, has no significant effects despite its dominance in the cases with few agents (in 53.45% and 42.60% of the iterations for 3 and 4 agents the winner is hyper-efficient and only for 8.54% of the iterations for 60 agents). Now given that in the Hybrid auction the winner receives the DEA based payment only if it is less than the second score payment, these results suggest that the VRS technology introduces an improvement, albeit not a significant one, for that specific value of the parameter α, since for the vast majority of the cases the Hybrid VRS auction payment to the winner is equal to the second score auction. On the contrary, for the Hybrid CRS auction the percentage of cases whereby the second score payment is lower than the DEA CRS payment decreases from 39.6% to 15.2% as the number of agents increases to 60 (Figure 2 : Plot d), with hyper-efficiency non existing.

For the second part of these simulations we fix the number of participating agents to 60 and begin to examine the dependence of the expected payments on the concavity of the principal's value function. We measure the concavity, based on the Arrow-Pratt coefficient which for the particular value function used in these simulations is equal to α. We follow an identical process by plotting the expected payments for the winners of both VRS and

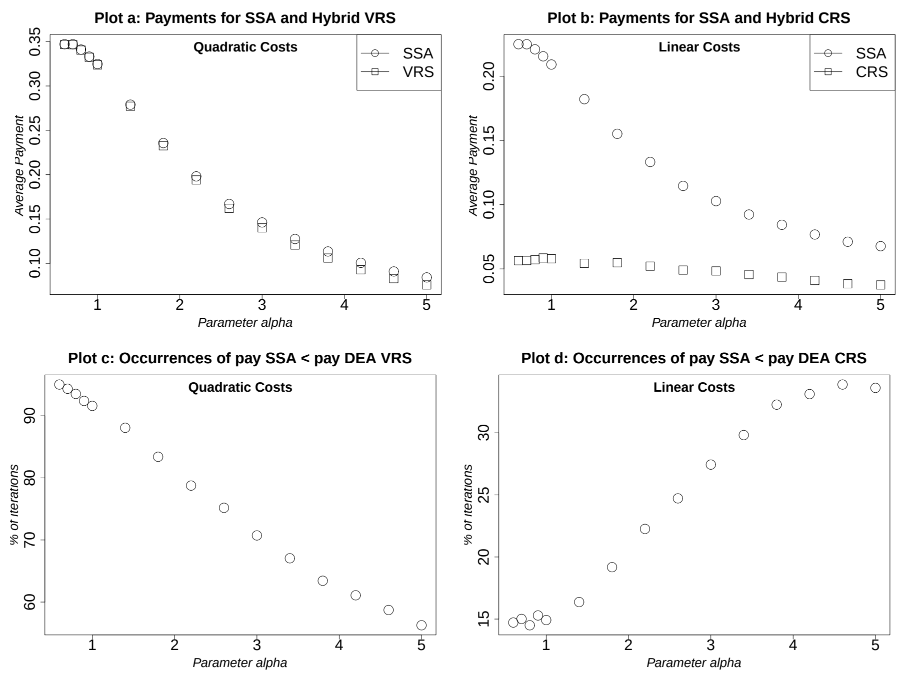

Figure 3: Single dimensional qualities for $\alpha = \{0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.4, 1.8, 2.4...5\}$ and $n = 60$ agents.

CRS Hybrid auctions and second score auctions (Figure 3: Plots a and b) for α in $\{0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.4, 1.8, 2.4...5\}$ and then the percentage of occurrences of the second score payment being lower than the DEA VRS and CRS payments (Figure 3: Plots c and d).

In more detail, in Figure 3 : Plot a we show that, as in the previous simulations, the average Hybrid VRS payment is very close to the second score auction payment, while for linear costs, the average SSA payment is again higher than the average Hybrid CRS payment. However, as opposed to previous simulations, there is a clear indication that as α increases, average Hybrid VRS payment is in-fact less than the average second score payment. The intuition behind this result, is that as the principal's risk aversion increases, the utility function becomes more and more curved in $(0, 2]$ (the space of agents' qualities). Consequently, the score function gets less power since it will tend to envelop the points less closely. A secondary result is that all (second score and both Hybrid auctions) average payments decrease as the parameter α increases for $\alpha >= 0.7$. This is to be expected, since a heavy risk averse principal (high value of parameter α), will favour lower qualities which result in lower payments.

Finally, regarding the occurrence of cases in which the SSA payment is less than the VRS DEA payment, the decrease is almost linear as the parameter α increases (Figure 3 : Plot c), while for the linear cost functions the occurrence of cases where SSA payment is less than the CRS DEA payment increases as the parameter increases (Figure 3 : Plot d). Both results justify our initial findings since the increase of the gap between the SSA and Hybrid VRS payment suggests that the number of cases where the Hybrid auction payment is equal to the second score one decreases, while exactly the opposite happens for the Hybrid CRS payment which becomes more equal to the second score as α increases.

4.2. Multi-dimensional output

In this set of simulations we consider three dimensional qualities, hence we adjust the principal's value function to $V(y) = 4(3 - e^{a_1 y_1} - e^{a_2 y_2} - e^{a_3 y_3})$ with $y_1, y_2, y_3 \sim U(0, 2)$. Likewise, we adjust the agents' cost functions to $x(y) = c_1 y_1^2 + c_2 y_2^2 + c_3 y_3^2$ in the quadratic case and to $x(y) = c_1 y_1 + c_2 y_2 + c_3 y_3$ in the linear case, with $c_1, c_2, c_3 \in (0, 1]$.

We follow a similar process to the one for single-dimension qualities and calculate the payment the winner gets in the second score auction and the Hybrid VRS and CRS auctions (Figure 4 : Plot a and b), with the number of the participating agents varying from 3 to 60. In terms of the ordering of the average payments we receive similar results as in the first set of simulations. In addition to that, the appearance of a hyper-efficient winner almost follows the pattern of the single-dimensional simulations i.e. for the Hybrid VRS there is a hyper-efficient winner at 87.5% and 81.9% of the iterations for 3 and 4 agents and 33.8% for 60 agents.

In the single-dimension counterparts the occurrences of the cases where the SSA payment is lower than the DEA VRS payment decreased monotonically with the number of agents. In the multiple dimension case, this does not happen, cf. Figure 4 : Plot c (monotonicity exists for the percentage of SSA payment less than he DEA CRS ones cf. Figure 4 : Plot d).

Finally, for any auction the expected payment to the winner is higher in the multi-dimensional case than in the single-dimensional case. Indeed, the introduction of 3 dimensions results in an increase of the average second score payments in a range of 2.48 to 5 times higher than the single-dimensional case for 3 to 60 agents. The same pattern appears for the linear cost case, where that ration is ranging from 2.8 to 7.4.

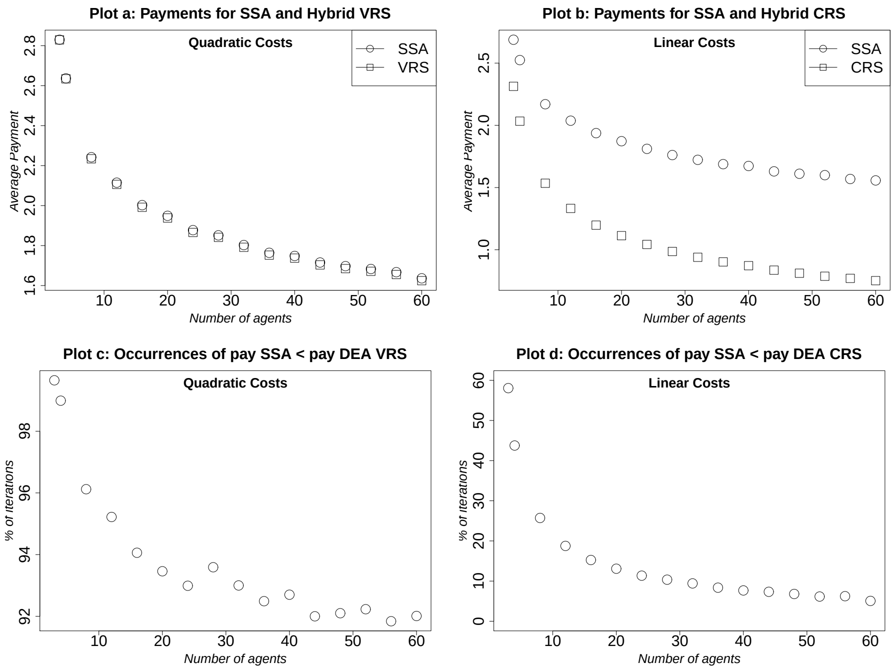

Figure 4: Multi dimensional qualities for $n = \{3, ..., 60\}$ agents, $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and $c_1, c_2, c_3 \sim U(0, 1)$.

5. Conclusions

To sum up, we have shown that for both single and multi dimensional qualities, the use of a combination of benchmarking and second score thinking lowers the expected payment to the providers, and more so, the stronger assumptions (linear, quadratic) we impose on the underlying cost function. The VRS DEA technology does not provide a significant increase in the efficiency of the second score multi-dimensional auction on behalf of the principal since it results in almost identical payments to the winner. Indeed, for the few iterations in which it is preferable for the principal to select the DEA payment over the second score payment, there is a small improvement. This is highlighted in the single-dimensional qualities simulations, where the ratio between the Hybrid VRS and the second score average payment is always less but very close to 1, and the Hybrid VRS payment is equal to the second score one for 82.9% to 95.25% of the simulation's iterations. This suggests that the relative small number of iterations in which the DEA VRS payment is less than the second score payment is enough to push the average ratio of Hybrid VRS to SSA below 1.

In addition to that, we showed that the ability of the score function to limit payments to the provider depends on the concavity of the score function. As the principal's utility function gets more concave, i.e. as he gets more risk averse with a higher value of α, the role of the DEA benchmarks becomes more important. The intuition is that the more curved score function gives a score based approximation more similar to the DEA FDH model where we only impose free disposability of inputs and outputs.

We also showed how the number of bidding agents impact the outcome. In general, more agents will make both the SSA and the DEA based payments lower. In particular for a low number of bidders, extra bidders will have a large marginal impact on the payments. This suggests that the procuring principal should make an effort to engage more bidders.

Lastly, we showed that the introduction of additional output / quality dimensions significantly increases the expected payment. We are aware that this may not be a surprising result since now the average costs and principal's value function are higher from the single-dimension case since three outputs are produced. However, this increase can be also be attributed to fact than now the ability of DEA to approximate the cost function and the power of the the second score principle to limit the payments are both undermined by the extra dimensions. That is, with more dimensions quite a few extra bidders are needed in order to span the cost function with a given precision. This suggests that the principal should think carefully on which qualities really matter with an attempt to limit these effects.

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