A Sealed-bid Two-attribute Yardstick Auction Without Prior Scoring*

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Abstract

We analyze a two-attribute single item procurement auction that uses yardstick competition to settle prices. The auction simplifies the procurement process by reducing the principal's articulation of preferences to simply choosing the most preferred offer as if it was a market with posted prices. This is done simply by replacing the submitted sealed bids by yardstick bids, computed by a linear weighting of the other participants' bids.

We show that there is only one type of Nash equilibria where some agents may win the auction by submitting a zero price-bid. Using a simulation study we demonstrate that following this type of equilibrium behavior often leads to winner's curse. The simulations show that in auctions with more than 12 participants the chance of facing winner's curse is around 95%.

Truthful reporting, on the other hand, does not constitute a Nash equilibrium but it is ex post individually rational. Using a simulation study we demonstrate that truthful bidding may indeed represent some kind of focal

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1. Introduction

Efficient and flexible procurement systems are often crucial for the success of any organization. The procurement system should enhance competition on both price and quality while being as simple as a posted price market. In this paper we analyze a two-attribute auction mechanism aiming at this level of simplicity. The mechanism uses the so-called yardstick competition in order to both facilitate price competition across multi-attribute offers and to postpone the buyer's procurement decision to simply select the most preferred commodity or service as in a traditional market with posted prices. The applied yardstick principle is known from the regulation of natural monopolies (see e.g. [1]) and [8] has previously suggested yardstick competition as an integrated part of multi-attribute auctions. However, as [8] point out, straightforward use of yardstick competition introduces sophisticated strategic manipulation. We analyze this in more depth and show that this rather rough but simple straightforward use of yardstick competition promotes truth-telling and makes complex procurement look like posted prices. As such, this paper is in line with the ongoing challenge in designing operational flexible trading systems that facilitate transparent competition on both price and other attributes - while keeping the transaction costs low.

To be more precise we consider a procurement mechanism where: A group of sellers (with private information about their production cost) each submit a sealed price-quality bid, to be interpreted as the quality level they are willing to deliver if compensated by at least their asking price. The sealed price-bid is replaced by a *yardstick price* which is determined as a convex combination of the two efficient "neighbor" price-bids. The buyer then selects one of these bids as a winning bid (without having to articulate his preferences, via a scoring function, in advance). The winner commits to deliver and is compensated with his associated *yardstick price*.

As such, this mechanism is a special case of what is called a Data Envelopment Analysis (DEA) based auction in a recent paper by [8]. Although they consider a mechanism with known scoring function, their paper already noted that when the buyer's preferences are unknown to the sellers, their strategies become complicated to analyze and that they are likely to deviate from truth-telling by bidding above their true cost.

In the present paper we follow up on these conjectures. We show that there are no Nash equilibria where all sellers submit a strictly positive price-bid. For instance, the seller with the highest quality level can always win the auction by submitting a zero price-bid and being reimbursed the computed yardstick price. Yet, bidding zero may prove to be a fatal strategy since the seller will often win the auction with a loss. Using a simulation study we show that in auctions with 4 participants the chance of winning the auction with a gain for the seller with the highest quality level is only 20% and that this number quickly drops to 5% as the number of participants increases. In other words, it appears that equilibrium behavior in the sense of zero quantity-price bidding is highly risky for the sellers who can easily end up facing winner's curse if they follow such a strategy.

On the other hand, we shall argue that even though truth-telling is not a Nash equilibrium it is still very likely to be some kind of a focal point when using the mechanism in practice. The basic intuition is that since improving a seller's chances of winning the auction requires bidding quite substantially above or below the true cost, the sellers run into two problems: a) bidding substantially above increases the risk of being excluded from the auction (in the sense that the bid lies above the yardstick price) and b) bidding substantially below increases the risk of winning the auction with a loss since the compensation is likely to be below the actual true cost.

In our simulations, we consider the bidder who wins the auction if everyone tells the truth and then we examine if this bidder will remain the winner even if we allow the other bidders to misreport their true cost with up to 20% both above and below. We show that allowing the other bidders to misreport rarely results in a new winner that wins with a gain. It happens in less than 11-20% of all cases. Combined with the fact that truth-telling guarantees non-negative pay-offs (is ex post individually rational) our results point towards truth-telling as a focal point in practice.

We emphasize that this paper is *not* about designing the optimal procurement auction, but rather an attempt to analyze the performance of this particular multi-attribute yardstick auction inspired by the DEA-based auction in [8]. The reason for looking at yardstick prices as opposed to any other type of posted prices in the form of mark-ups on the observed price-bids, is that the sellers are unable to influence their own yardstick price by their bid. Another reason is that the construction of the yardstick prices does not involve the buyers weighting prices and other attributes. Hence it becomes apparent that the yardstick auction may have some desirable strategic properties: Indeed, as our paper demonstrates, this is highly likely to be the case in practice.

As mentioned, we focus specifically on the situation where it is too costly (or impossible) for the buyer to articulate his preferences, e.g., via a scoring function. It is well recognized that preference elicitation in e.g. multi-attribute auctions is costly and requires substantial decision support or mechanism design that mitigate these costs, see e.g. [27], [21] and [22]. Determining a scoring function may be complicated for several reasons: For instance, when the buyer is a single person and the scoring represents the buyer's intra-personal trade-offs between price and quality; these complications are a central topic of a large literature on Multiple Criteria Decision Making (MCDM), see e.g. [26]. Furthermore, when the buyer represents a group of persons (e.g., an organization), the construction of the scoring function may further involve inter-personal conflicts[2]. The complication of this is reflected on the large literature on Social

[2]For instance think of a procurement decision in a cooperative where different members' interests have to be aggregated

Choice, see e.g. [3] and [20].

Empirical cases support the claim that determination of a scoring function is a difficult matter. For instance, in the conservation reserve program the USDA (United States Department of Agriculture) ranks the bids into a score. The actual determination of these scoring rules has been widely discussed, see e.g. [5, 4]. The applied scoring has also been an issue in the wholesale market for electricity in California, where the choice of an unsuitable scoring rule had severe consequences, see [12] and [13]. Hereby, we conclude that if the cost of constructing suitable scoring functions is too high, there are alternative types of procurement mechanisms that leave out a priori preferences and hereby potentially make the entire procurement simpler.

The outline of the paper is as follows: Section 2 relates the paper to the existing literature and Section 3 introduces the procurement setting and the notion of yardstick prices. Section 4 defines the yardstick auction and discusses strategic bidding by the sellers. Section 5 introduces the simulation framework and the results are presented in Section 6 along with a discussion of our parameter choices. Section 7 concludes.

2. Relation to the literature

Unlike auctions in general, the theoretical literature on multi-attribute auctions is relatively sparse. A related line of literature, however, concerns the widely used systems for e-procurement which have several similarities with multi-attribute auctions, for example in how they automate negotiations (see e.g. [11] for an introduction to some of these systems). There are several papers suggesting an incorporation of multi-attribute auctions into the so-called Request for Quote (RFQ) systems, see e.g. [19] and [7]. RFQ systems use the Internet to improve the searching and matching process between buyers and sellers in general. [18], [24] and [25] provide surveys as well as suggest mechanisms that combine multi-attribute negotiation and auction systems.

The seminal paper on multi-attribute auctions by [14] analyzes the two most common scoring auctions: the *first score* and the *second score auction*.[3] These two auctions have many similarities with the first and second *price* auctions. In fact, [14] proves that the *revenue equivalence theorem* also holds for the first and second score auctions[4] and shows that the second score auction is efficient and strategy proof.[5]

In the literature on multi-attribute auctions there are only a few papers relaxing the assumption of an *a priori* given value function for the principal. [15] investigate the issues of setting quality thresholds that are unknown to the bidders. [6] study the sequential learning of the value function and bidders' cost functions by a sequence of scoring auctions with different scoring functions. However, [6] do not directly address the risk of strategic bidding and in essence presume truthful revelation in the sequence of trial auctions.

In this paper we analyze a yardstick auction which basically replaces the principal's scoring with a yardstick competition: The principal simply chooses among "posted" yardstick prices. As a starting point for this research we use the paper on DEA auctions by [8]. However, we relax their assumption that the principal will announce *a priori* his scoring function[6].

3. The model

We consider a procurement setting along the lines of the model in [14]. A risk neutral principal is seeking to procure a commodity (or a service) from one of n risk neutral agents, $i \in N = \{1, ..., n\}$. The commodity supplied by agent i is characterized by a one-dimensional quality level $y^i \in \mathbf{R}_+$. In order to focus on the adverse selection problems we further assume that delivery of the promised qualities can be costlessly enforced (e.g. by a harsh penalty for deviations).

[3] In a first score auction, the bidder with the highest score wins and has to meet the highest score. In a second score auction, the bidder with the highest score wins and has to meet the second highest score.

[4] Using the revenue equivalence theorem as it is presented in [23].

[5] However, it is not given that the second score auction is the most preferred auction by the principal. [8] show that it is possible for the principal to extract more informational rent while the auction remains efficient and strategy-proof.

[6] The idea of a yardstick auction can also be found in [2]. They suggest an auction design for so-called combinatorial auctions based on the same type of yardstick principle.

Agents (sellers) can produce different levels of quality and their costs will depend on an underlying common cost function as well as their individual efficiency levels. More formally, an agent producing the quality level y^i faces a cost $c^i(y^i, \epsilon^i) \in \mathbf{R}_+$. The parameter ϵ^i represents the efficiency of agent i which is private information. Meanwhile, all agents know that they operate under a common cost structure $C(y) \subseteq \mathbf{R}^{\mathbf{2}}_+$.

Regarding this common cost structure, we assume that

$$C(y) = \min\{x \mid y \text{ can be produced at cost } x\}$$

satisfies $A1$ and $A2$ below:

$A1$. C(.) is weakly increasing : $y' \geq y \Rightarrow C(y') \geq C(y)$,

$A2$. C(.) is convex : $C(\gamma y + (1-\gamma)y') \leq \gamma C(y) + (1-\gamma)C(y'), \forall \gamma \in [0,1]$.

We will think of agents efficiency in terms of the classical radial Debreu/Farrell efficiency index (see e.g., [16]). That is, the individual cost of agent i is determined as

$$c^i(y^i, \varepsilon^i) = \varepsilon^i C(y^i). \qquad (1)$$

and the bids can be interpreted as observations of the underlying common cost function. The principal is unaware of $C(\cdot)$ but knows that C satisfies A1 and A2.

We assume that the actual quality level y^i is verifiable and fixed for each agent i. Thus, possible strategic manipulations by the agents can only regard the costs, and the signal from each agent i is simply a price-quality bid

$$(x^i, y^i) \in \mathbf{R}^{\mathbf{2}}_+ \qquad (2)$$

with the interpretation that agent i will produce his quality level y^i if he is paid at least x^i.

The principal uses the bids to determine which agent to procure from and what to pay him and the other agents through an auction mechanism. To formalize, let $d^i(x^i, y^i) \in \{0,1\}$ indicate whether agent i supplies the offered commodity or not and $f^i(x^i, y^i) \in \mathbf{R}_+$ denote his payment. As it will become clear below, $f(x^i, y^i)$ depends on the agent i's yardstick price (defined in Section 3.1).

The aim of the participating agents is to maximize their (expected) profit. The profit to the winning agent is:

$$\pi^i(x^i, y^i) = f(x^i, y^i) - c^i(y^i, \epsilon^i), \tag{3}$$

while the aim of the principal (buyer) is to maximize (expected) net private value, i.e., the value generated by the good minus the compensation to the agent. We follow the literature in assuming that value is represented by a weakly increasing concave function of the agents' qualities $V(y^i)$. However, it should be noted that the principal is unaware of the value of the good V simply because it may be too costly to assess all the information needed to express a full value function. However, when facing a list of posted prices, the principal is able to make a specific choice.

3.1. The Yardstick Prices

From the number of different cost models satisfying requirement $A1$ and $A2$, we follow [8] and focus on a model where the cost structure (illustrated in Figure 1) is estimated using the smallest convex envelopment of the observed bids $\{(x^j, y^j)\}_{j \in N}$, i.e.,

$$\begin{aligned}\widehat{C}(y) = \inf\{x \in \mathbf{R}_+ \mid x \geq \sum_{j \in N} \lambda^j x^j, \quad y \leq \sum_{j \in N} \lambda^j y^j,\\ \sum_{j \in N} \lambda^j = 1, \lambda^j \geq 0, \forall j \in N\}.\end{aligned} \tag{4}$$

For each agent $i \in N$ we define a *yardstick price* $\bar{x}^i$ using the estimated cost structure (4) on the reduced bid-sample where agent i's bid is excluded (illustrated in Figure 2):

$$\begin{aligned}\bar{x}^i = \inf\{x \in \mathbf{R}_+ \mid x \geq \sum_{j \in N \setminus \{i\}} \lambda^j x^j, \quad y^i \leq \sum_{j \in N \setminus \{i\}} \lambda^j y^j,\\ \sum_{j \in N \setminus \{i\}} \lambda^j = 1, \lambda^j \geq 0, \forall j \in N \setminus \{i\}\}.\end{aligned} \tag{5}$$

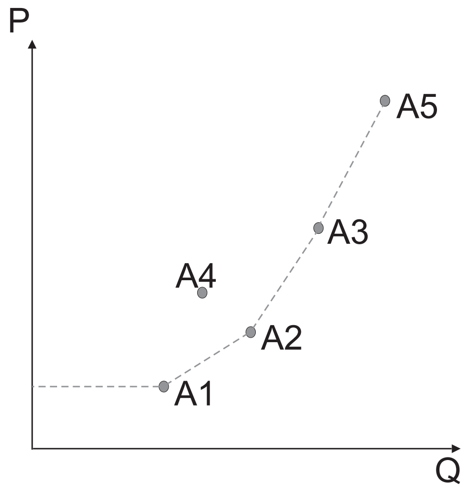

Figure 1: Convex envelopment of the submitted bids

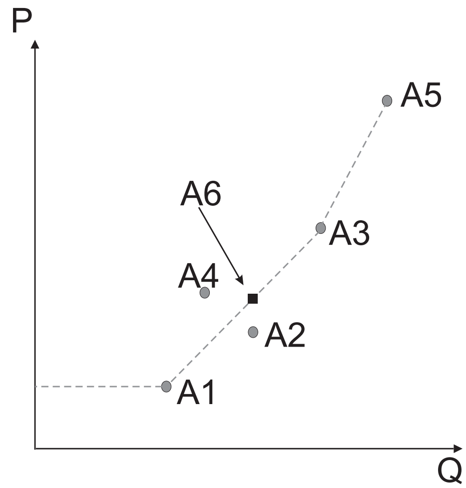

Figure 2: The yardstick price for agent 2

For agent i, the above solution identifies a single point $(\bar{x}^i, y^i)$ on the frontier estimated by the smallest convex envelopment of the submitted bids except for agent i's own bid.

Note that the bid of agent i has no influence on his own yardstick price, but influences the yardstick price of the neighbor bids if his bid is cost efficient. Let $N^e \subset N$ denote the set of agents with cost efficient bids. Agent $i \in N^e$ has a neighbor to the left (i^l) and to the right (i^r) defined as,

$i^l = \arg\max_{j \in N^e}\{y^j | y^j < y^i\}$

$i^r = \arg\min_{j \in N^e}\{y^j | y^j > y^i\}$.

For example, in Figure 2 the left neighbor of agent 2 is agent 1 and the the right neighbor is agent 3. Thus, 2's bid influence the yardstick prices of both agent 1 and 3.

Furthermore, note that for the bidder with maximal value of y, the associated yardstick price will be infinite. Thus, in the yardstick auction presented below we require the principal to be able to announce an upper bound on the bids, i.e. the highest value of y and its associated reservation price x that the principal is willing to accept so that every agent is guaranteed a yardstick price. We denote this reservation point z^P. Consequently, there is no need for a specification of a similar lower bound on quality.

We find it reasonable to assume that the principal is able to specify z^P, or in other words that the principal can express the willingness to pay for the best possible quality and only that. Although we consider this limited articulation of preferences to be unlike the difficulties in e.g. expressing aspiration points in multi-criteria models as discussed in [17], we recognize that expressing z^P is associated with a cost and leave it for future research to find a way to avoid such preference information a priori. As an example, a principal who lacks information about high quality may exclude bids, but in practice such cases will occur with very low probability; and if it does happen, the principal can adjust his reservation quality and price over time when/if the auction is repeated. As such z^P could be the result of a learning process for the principal.

Example 1: Let four agents $\{1, 2, 3, 4\}$ submit bids:
$z^1 = (x^1, y^1) = (1, 3)$,
$z^2 = (x^2, y^2) = (2, 5)$,
$z^3 = (x^3, y^3) = (3, 6)$,
$z^4 = (x^4, y^4) = (2.5, 4)$,

and let the principal state his reservation point $z^P = (5, 7)$. Note that agent 4's bid is inefficient when the common cost function is estimated by the smallest convex envelopment of the four bids.

Now, according to the definition (5) the yardstick bids are found as follows:
$\bar{z}^1 = (\bar{x}^1, y^1) = (2, 3)$,
$\bar{z}^2 = (\bar{x}^2, y^2) = (2.33, 5)$,
$\bar{z}^3 = (\bar{x}^3, y^3) = (3.5, 6)$,
$\bar{z}^4 = (\bar{x}^4, y^4) = (1.5, 4)$.

Consider for example the determination of the yardstick price for agent 2. Agent 2 has efficient "neighbor" bids by agent 1 and 3. Hence, the yardstick is found from the convex combination $\lambda(3, 1) + (1 - \lambda)(6, 3)$ for $y = 5$, i.e., for $\lambda = 1/3$, yielding yardstick price $\bar{x}^2 = 1/3 \times 1 + 2/3 \times 3 = 2.33$.

Note that for agent 1 only the bid of agent 2 is used to determine the yardstick while for agent 3 we need the reservation point z^P as the right "neighbor" bid (with z^2 as the left bid). Moreover, since the bid of agent 4 is inefficient the yardstick price $\bar{x}^4$ is smaller than the original price-bid x^4. □

4. The Yardstick Auction

We analyze a procurement auction defined by a stepwise procedure. In Step 0, the principal starts the auction by publicly announcing z^P stating the maximum value of the attribute in question y^P and its reservation price x^P for y^P. z^P enters the auction as a submitted bid and thereby, x^P addresses the problem of non-existing yardstick price for the maximal value of y among the bidders as explain in Section 3. Then, in Step 1, the bidders submit sealed bids. In Step 2, the yardstick prices ($\bar{x}^i$) are computed (as defined in 5 and illustrated in Figure 2). If $\bar{x}^i \geq x^i$ the bidder's original price-bid is replaced by the computed yardstick price otherwise the bidder will not be assigned a yardstick price (i.e. a cost inefficient bidder will not be assigned a yardstick price). In Step 3, the principal reviews the yardstick bids and selects a single yardstick bid as the winner of the auction. Step 4 finalizes the auction by compensating the selected winner with his yardstick price. Formally,

The mechanism:

Step 0: The principal announces the procurement proposal and an upper bound on the bids $z^P = (y^P, x^P)$, where y^P is the highest acceptable value of y and x^P is the highest acceptable price for y^P.

Step 1: Each participant $i \in N$ submits a single sealed bid $z^i = (x^i, y^i)$. Let Z be the set of bids including z^P, i.e., $Z = \{z^i\}_{i \in N} \cup z^P$.

Step 2: A yardstick price $\bar{x}^i$ for all $i \in N$ is computed using (5) and replaces x^i if $x^i \leq \bar{x}^i$. If, for some $j \in N$, $x^j > \bar{x}^j$, agent j will not be assigned a yardstick bid. Let $\bar{z}^i = (\bar{x}^i, y^i)$ be the yardstick bid of agent $i \in N$ and let $\bar{Z}$ be the set of such yardstick bids.

Step 3: The set $\bar{Z}$ is presented to the principal, who selects the winning bid $\bar{z}^{i^*}$

Step 4: Only i^* is compensated by $\bar{x}^{i^*}$.

Let $\succ$ on $\bar{Z}$ denote the principal's binary preference relation. Following the notation introduced in Section 3 the auction is defined by the allocation and payment functions which take the following forms:

$$d^i(z^i) = \begin{cases} 1 & \text{if } \ \bar{z}^i \succ \bar{z}^j \text{ for all } \bar{z}^{j \neq i} \in \bar{Z} \\ 0 & \text{if otherwise} \end{cases} \tag{6}$$

$$f^i(z^i) = \begin{cases} \bar{x}^i & \text{if } \ \bar{z}^i \succ \bar{z}^j \text{ for all } \bar{z}^{j \neq i} \in \bar{Z} \\ 0 & \text{if otherwise} \end{cases} \tag{7}$$

where $z^i = (x^i, y^i)$.

To this end, the auction may be seen as a mechanism for settling posted prices on services or commodities with linear weighting of price and other attributes. In fact in comparing with the second score auction with linear weighting, the mechanism settles the most pessimistic prices seen from the principal's point of view. To see this, note that the yardstick prices are equal to the highest possible second score compensation with linear scoring.

4.1. Bidders' Strategic Behavior

We now turn towards the bidders' strategic behavior. As discussed, we consider the situation where it is impossible (or very costly) for the principal to articulate a scoring function *a priori*. For instance, in case of a public institution that represents social (aggregate) preferences. However, we assume that it is possible for the principal to make a unique selection in Step 3 of the mechanism.

The fact that the principal cannot announce (and commit to) a scoring function *a priori* complicates the analysis of the bidders' strategic behavior simply because bidders cannot form an expectation concerning how their bid will influence the chance of winning the auction. Indeed, bidders only know their cost c^i and their quality level y^i.

Bidders choose their price-bids x^i strategically and want to maximize their expected pay-off $\pi^i(x^i, y^i) = \bar{x}^i \times \text{Prob}\{i \text{ wins}\} - c^i(x^i, y^i)$. Note that $\bar{x}^i$ and c^i are unaffected by i's strategic choice of price-bid. Thus, i can only influence his expectation of winning the auction Prob$\{i$ wins$\}$.

We say that a strategy profile $\{\tilde{x}^i\}_{i \in N}$ is a *Nash equilibrium* if there does not exist an agent j and a strategy x^j such that $\pi^j(x^j, y^j) > \pi^j(\tilde{x}^j, y^j)$ given $\{\tilde{x}^i\}_{i \neq j}$.

Although, agents do not know how the principal will choose the winner of the auction and cannot influence their own yardstick price there is still ample room for manipulation.

Observation 1: *Consider a given agent $i \in N$. By increasing his price-bid x^i up to at most the yardstick price $\bar{x}^i$ both neighbor yardstick bids increase and so does agent i's expectation of him winning the auction.*

The argument is straightforward: Since changes in agent i's price-bid x^i have no influence on i's computed yardstick price $\bar{x}^i$ (as this is determined excluding agent i's bid from the data set), it is free for agent i to increase his price-bid up to his yardstick price. Bidding his yardstick price, given the bids of the other agents, increases the yardstick-price of agent i's neighbors and thereby increases his chance of being selected by the principal. Note that if agent i bids above his yardstick price-bid he will loose the auction for sure.

In a similar fashion we can show that if agent i decreases his price-bid, both neighbor agents will get decreasing yardstick price bids which in turn decrease i's expectations of winning the auction. Thus, bidding below one's true bid is disadvantageous *unless* the bid is so low that it in effect excludes the neighbor agent from the auction (in the sense that it makes the neighbor agent j's yardstick bid go below j's price-bid). Such a situation is illustrated in Figure 3, where agent i decreases his bid z^i to z'^i and hereby excludes j from the auction.

We record this by the following observation.

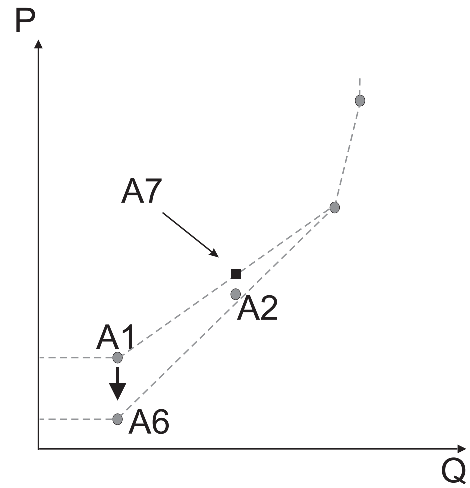

Figure 3: Excluding the neighbor yardstick bid

Observation 2: *There may be situations where an agent by bidding sufficiently below his true cost can exclude a neighbor agent's bid and thereby increase the expectation of him winning the auction*

As indicated by Observation 2, there may exist Nash equilibria where some agent (say, the one with highest quality level) excludes the other agents by a 0-price-bid.

Example 2: Consider three agents $\{1, 2, 3\}$ with true cost-quality combinations; $(c^1, y^1) = (1, 2), (c^2, y^2) = (2, 4)$ and $(c^3, y^3) = (3, 7)$. Assume that the principal decides on $z^P = (6, 8)$. Now, let agents 1 and 2 submit their true bids $z^1 = (1, 2)$ and $z^2 = (2, 4)$ while agent 3 submits the bid $z^3 = (0, 7)$. Clearly, the profile $z = (z^1, z^2, z^3)$ is a Nash equilibrium since yardstick prices become $\bar{x} = (0, 0, 5)$. Therefore the principal is presented with the singleton set of yardstick bids $\bar{Z} = \{(5, 7)\}$ which he then chooses as winner independent of his preferences. Hence, agent 3 is optimizing and agent 1 and 2 cannot do better given the strategy of agent 3. Consequently, the strategy profile z is a Nash equilibrium. $\triangle$

The example above reveals the existence of Nash equilibria where the k'th agent (ordered according to quality level) submits a 0-price-bid while agents with higher quality levels submit the truth and agents with lower quality levels can submit any bid. What determines the number k is whether the k'th agent has a yardstick price, given a 0-price-bid of agent $k-1$, which is above k's true cost such that he does not win with a loss: by bidding the truth, agents $k+1$ to n ensure that they do not win with a loss. Agents 1 to $k-1$ are in effect excluded from the auction by agent k's 0-price-bid.

In fact, such 0-bid equilibria are the only type of Nash equilibria in the model. Indeed, we can show that there does not exist equilibria where all bidders submit a positive price-bid. We record this as Observation 3 below.

Observation 3: *No Nash equilibrium exists for which $x^i > 0$ for all $i \in N$.*

Proof: By contradiction assume that an equilibrium exists for which $x^i > 0$ for all $i \in N$. By Observation 1, no agent will bid below his yardstick price: Indeed, if some bid is below the yardstick price the agent could increase his chances of winning by increasing his bid up to the yardstick price. Also bids cannot be above the yardstick price since this would lead to exclusion and thereby zero pay-off. Thus, all bids must lie on a horizontal line where $x^i = x^P$ for all $i \in N$: Indeed, if $x^j \neq x^P$ for some j then there exists an agent h for which $y^h \geq y^j$ and $\bar{x}^h \neq x^h$.

When $x^i = x^P$ for all $i \in N$, the agent i^* with the highest quality level y^{i^*} can win the auction by bidding $(y^{i^*}, 0)$ and be compensated with the yardstick price $\bar{x}^{i^*} = x^P \geq x^{i^*}$ contradicting that the strategy profile is a Nash equilibrium. Q.E.D.

An immediate consequence of Observation 3 is that truth-telling cannot constitute a Nash equilibrium. In practice, 0-price bidding is a risky strategy though. Often it will lead to winner's curse as we demonstrate by a simulation study in the next section. In fact, we also demonstrate that truthful bidding may well turn out to be a focal point in practice, also because it is ex post individually rational.

5. Simulation Framework

Based on the Yardstick Auction described in the previous section, we now introduce the simulation framework that allows us to analyze two scenarios: a) 0-price Nash equilibrium behavior of the bidder with highest quality level, and b) truth-telling as an alternative focal point.

In the proposed framework the principal's value function is given by $V(y, \alpha) = \alpha y$, with α independently drawn from the uniform distribution $\mathcal{U}(6, 16)$ and y the agents' reported quality levels, independently drawn from the uniform distribution $\mathcal{U}(1, 10)$. Most importantly, drawing parameter α from a random distribution, models the principal's uncertainty of its preference function before receiving the agents' bids.

Furthermore, the agents' costs are determined by the common underlying cost function $x(y) = y^2$ and the individual inefficiencies in production modeled by the parameter $\epsilon^i \sim \mathcal{U}(1, 1.5)$ resulting in individual costs $x^i(y) = \epsilon^i y^2$.

We simulate the mechanism 10^3 times for $4, 8, 12, 16, 20, 24, 28, 32$ participating agents. In every iteration we simulate for each one of the agents a set of bids (y^i, x^i) by randomly drawing y^i and ϵ^i and the principal's preference by randomly drawing α. For every iteration there is also an upper bound bid $z^P = (y^P, x^P)$ with y^P being equal to the upper bound of the distribution of the reported quality (hence $y^P = 10$) and $x^P \sim U(90, 110)$. We compute the yardstick bids and corresponding scores, identify the winner of the auction and calculate the utility that the winner of the auction derives after producing the promised quality. We then introduce deviation from truth-telling for all agents but one (labeled as the 'selected agent') by multiplying the agents' costs with a parameter randomly drawn from a uniform distribution centered in 1. For a deviation up to X% above and below the true cost we use $\mathcal{U}(1 - X, 1 + X)$. For example, a deviation up to 20% above and below the true cost we use $\mathcal{U}(0.8, 1.2)$.

Identifying the 'selected agent' depends on the scenario. Specifically, for the 0-price Nash equilibrium case the selected agent is the agent with the highest reported quality, while in the truth-telling scenario we focus our analysis on the agent identified as the ‘winner’ of the auction in a trial run where everyone reports the truth.

Assuming that all agents, except for the initial winner, are capable of under or over reporting their costs we proceed to compute their yardstick bids based on these ’misreported’ bids, and examine whether the selected agent in each scenario remains a winner despite the misreporting of the others.

Technically, all simulations are done in R and all DEA programs are solved using the ”Benchmarking” package for R, cf. [9] and [10]. Our parameter choices will be further discussed in section 6.1 below.

6. Simulation Results

Having described the simulation’s input parameters and objectives we now present our numerical findings grouped into two sets of simulations. First, we present the simulation results for the 0-price Nash equilibrium behavior of the bidder with the highest quality level and then, we present the simulation results of truth-telling as focal point.

The simulation results for the 0-price equilibrium scenario are reported in Figure 4. In Figure 4 (a) we show that as the number of participating bidders increases the percentage of auctions in which the bidder with the highest reported quality, denoted as i^*, wins the auction with a gain ($\pi^{i^*} \geq 0$) by bidding sufficiently below his true cost, drops significantly. The range of misreporting of all other agents, denoted as X, is equal to 10% and 20% (cu=0.1 and 0.2) respectively[7]. In particular, we see that for more than 12 bidders only approximately 5% of all auctions are successful for i^*. Moreover, as demonstrated in Figure 4 (b) the required misreporting of i^* approaches 100% (i.e. 0-price) proving that, in fact, aggressive bidding is needed to win the auction by ‘price-dumping’.

With the 0-price Nash equilibrium behavior being both highly risky for the manipulating bidder and potentially unrealistic in practice due to the close to

[7] Note that if the other bidders were allowed a higher degree of misreporting our results will be even stronger in the sense that there will be even fewer cases where the auction is won with a gain.

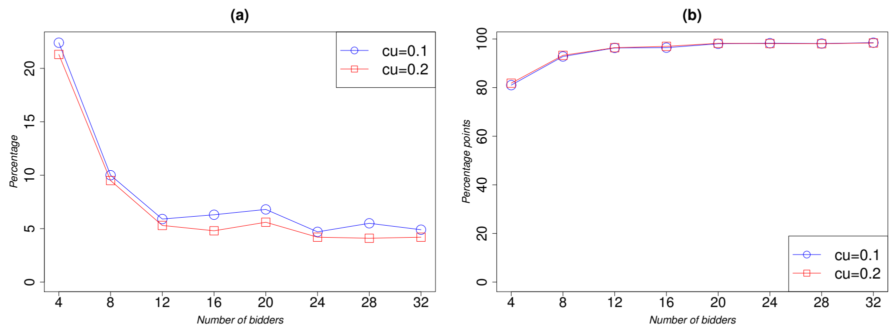

Figure 4: 0-price Nash equilibrium behavior: (a) the percentage of auctions won with a gain; (b) downward deviation in percentage of actual cost.

0-price-bids, we now turn to the simulation results concerning the truth-telling strategy. By definition, a bidder can not win the auction with a loss by telling the truth, however, other agents' misreporting may cause the otherwise winning bidder to lose the auction. The results from this simulation are reported in Figure 5. We fix the misreporting parameter X again to be between 0.1 and 0.2 and vary the number of participating agents. In Figure 5 (a) we show the percentage of auctions in which the initial winner remains the winner despite the fact that all other agents misreport. In Figure 5 (b) we focus on the cases where the other agents' misreporting results in a new winner (henceforth referred as "new-winner") and report the percentage of these cases where the new-winner wins with a loss.

The simulation indicates that for a reasonable degree of misreporting (up to 20%) the initial winner remains the winner in the vast majority of the simulation iterations despite the fact that all other agents misreport. Overall we consider this a positive result, especially given that both simulations also suggest that for the majority of the cases where misreporting results in a new winner, this new-winner faces a loss in utility. In fact in Figure 5 (b) we demonstrate that as the number of bidders increases, so does the percentage of new-winners facing losses in their utilities.

To sum up, simulations show that it requires a significant deviation to win,

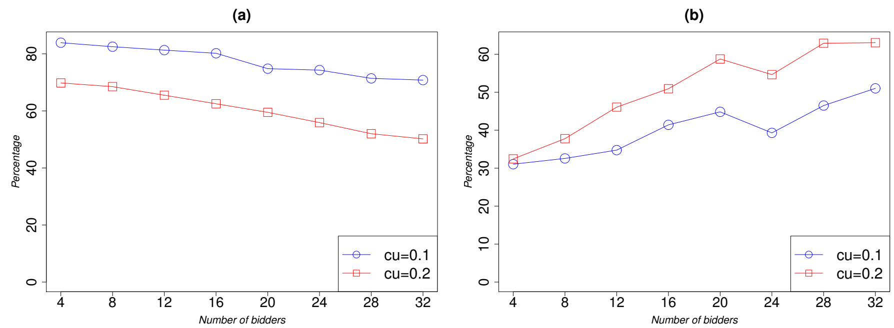

Figure 5: Truth telling as a focal point for the initial winner of the auction: (a) the percentage of auction where the initial winner remains the winner; (b) the percentage of new winners facing winner's curse.

while that significant deviation involves an increasing chance of winner's curse, i.e., that the winner wins with a loss. Combining the results from both scenarios shows that irrespective of the number of competing bidders as well as the degree of misreporting, 80 to 89% of all auctions will either have the same winner (the initial winner) or a new winner who wins with a loss.

Consequently, we conclude that bidding truthful may very well be a focal point in practice. It is ex post individually rational and the optimal strategy in the vast majority of cases.

6.1. Discussion of the simulation assumptions

In the simulation framework we make use of the uniform distribution in connection with various parameter choices. We shall here briefly discuss these choices and how they will influence the result of our simulation study.

Concerning the principal (or buyer) we have made two assumptions:

i) $\alpha \in \mathcal{U}[6, 16]$. The uniform distribution's limits for the parameter α have been set to 6 and 16 in order to represent a suitably broad range of potential preferences of the principal. Since we assume to have a common underlying cost function of the form $x(Y) = y^2$, truth-telling and no inefficiency in production (i.e. $\epsilon^i = 1 \; \forall \; i$) for all bidders, would imply that the principal picks a quality-price-bid where the quality level is between 3 and 8 (recall that $y \in \mathcal{U}[1, 10]$). Obviously more extreme quality levels can be selected when we allow for individual inefficiencies in production. Narrowing or spreading the interval $[6, 16]$ will have a negligible influence on our simulation results as confirmed by further simulations.

ii) $x^P \in \mathcal{U}[90, 110]$. The limits of the uniform distribution of x^P is determined as a plus-minus 10% deviation from the true underlying cost of 100. This reflects the principal's uncertainty when choosing z^P in the initial step of the mechanism. Unlike the choice of α, the choice of x^P has a direct impact on the simulation results in the sense that it influences the yardstick price of the agent with the highest quality level. As x^P increases so does the yardstick price of the bidder with the highest quality level. Hence, the higher the value of x^P the more it favors the 0-price bidding of the bidder with the highest quality level. By randomly drawing a level which is in a 10% range above and below true costs for $y = 10$ (maximum quality level) we therefore try to neutralize this effect.

Concerning the bidders (or sellers) we have made two assumptions:

i) $\epsilon^i \in \mathcal{U}[1, 1.5]$. That production units may have up to 50% technical inefficiency is supported by several empirical productivity studies (see e.g.[9]). The effect a change in this parameter can have on our simulation study is two-fold. On the one hand, higher inefficiency tends to exclude more bidders from the auction. On the other hand, increasing inefficiency tends to increase yardstick prices. Consequently, the probability of winning with a gain by following the 0-price strategy weakly increases with increased inefficiency level. Looking at truth-telling as a focal point the effect of changing the inefficiency level is far less obvious though. Further simulations tend to show that the effect is marginal.

ii) $x^i(y) = \epsilon^i y^2$. While the theoretical analysis is based on a more general class of convex cost functions defined in Section 3, in the simulations we consider quadratic cost functions mainly due to their popularity in auction related literature and the general economic theory. Simulations using other forms of cost functions such as $x^i(y) = \epsilon^i y^3$ confirms our previous findings.

7. Conclusion

We have analyzed a two-attribute procurement auction that uses yardstick competition to simplify the procurement process. The yardstick auction reduces the cost of articulating preferences to a mere problem of picking a favored alternative, as with posted prices.

Although the individual bidders cannot influence the compensation if winning, the auction is not strategy-proof. We proved that there only exist Nash equilibria which involve extreme 0-price bidding. Nevertheless, 0-price bidding behavior is clearly highly risky as illustrated by our simulations which showed that with sufficient competition among bidders the chance of facing winner's curse is around 95%. Truthful bidding on the other hand is ex post individually rational and as demonstrated by our simulations it is very likely that the bidder that wins if all bidders report the truth, remains the winner even if all other bidders are allowed to deviate substantially from the truth. Again it was shown that if deviation from the truth results in a new winner he is very likely to face winner's curse. This suggests, that in practice truthful reporting is an alternative focal point of the yardstick mechanism.

Assuming that the bidders report truthfully in practice it is possible to measure the cost of not investing in articulating a traditional "scoring function" for the principal. Additional simulations indicate that in the majority of cases the yardstick auction results in the same winner as a traditional second score auction with a priori announced scoring function. This confirms that the yardstick auction is not an efficient auction, however the simulations also indicate that the actual drop in social value from not having a scoring function decreases significantly. In fact, as the number of participants reaches approximately 10 there is only a marginal drop in social value from not having to articulate a scoring function.

Further adjustments of the auction set-up may strengthen the conjecture that truthful reporting is a focal point. For example we may try to limit bidding above true cost by using the principals choice to elicit potential scoring functions and use these along the lines of a traditional scoring auction. We leave the details for future research.

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